GCSE Maths – Algebra

Expressions Involving Surds and Algebraic Fractions

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving surds and algebraic fractions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Simplify $\sqrt{75a} + 8\sqrt{3} - 9\sqrt{3a}$

Step 1: Reduce all surds in the expression to their simplest form.

$$\sqrt{75a} = \sqrt{25 \times 3\,a} = \sqrt{25} \times \sqrt{3} \times a = 5 \times \sqrt{3} \times a = 5\sqrt{3a}$$

The other surds are in their simplest form.

$$\sqrt{75a} + 8\sqrt{3} - 9\sqrt{3a} = 5\sqrt{3a} + 8\sqrt{3} - 9\sqrt{3a}$$

Step 2: Simplify the expression using rules of surds.

Surds with the same value under the root can be added together or subtracted from each other.

$$5\sqrt{3a} + 8\sqrt{3} - 9\sqrt{3a} = 8\sqrt{3} - 9\sqrt{3a} + 5\sqrt{3a} = \mathbf{8\sqrt{3} - 4\sqrt{3a}}$$

Guided Example

Simplify $16\sqrt{7} + \sqrt{252} + 14\sqrt{98}$

Step 1: Reduce all surds in the expression to their simplest form.

$16\sqrt{7} = 16\sqrt{7}$

$\sqrt{252} = \sqrt{36 \times 7} = \sqrt{36} \times \sqrt{7} = 6\sqrt{7}$

$14\sqrt{98} = 14\sqrt{49 \times 2} = 14 \times \sqrt{49} \times \sqrt{2} = 98\sqrt{2}$

Step 2: Simplify the expression using rules of surds.

$16\sqrt{7} + 6\sqrt{7} + 98\sqrt{2}$

$= 22\sqrt{7} + 98\sqrt{2}$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Simplify the following expressions:

a) $\sqrt{78}d + 5\sqrt{13} - 19\sqrt{13}d$

None of these surds can be simplified

$d\sqrt{78}$

b) $6\sqrt{10}p - 9\sqrt{10} + \sqrt{40}p$

$p\sqrt{40} = p\sqrt{4\times10} = p\sqrt{4}\times\sqrt{10} = 2p\sqrt{10}$

$6p\sqrt{10} - 9\sqrt{10} + 2p\sqrt{10}$

$= 8p\sqrt{10} - 9\sqrt{10}$

$= \sqrt{10}(8p-9)$

c) $\sqrt{24}e - \sqrt{18}e - \sqrt{45}e + \sqrt{5}$

$\sqrt{24}e = \sqrt{4\times6}\,e = 2\sqrt{6}\,e$

$\sqrt{18}e = \sqrt{9\times2}\,e = 3\sqrt{2}\,e$

$\sqrt{45}e = \sqrt{9\times5}\,e = 3\sqrt{5}\,e$

$2e\sqrt{6} - 3e\sqrt{2} - 3e\sqrt{5} + \sqrt{5}$

d) $\sqrt{9}x + \sqrt{36}x - 20\sqrt{5}y + 5\sqrt{5}y$

$\sqrt{9}x = 3x$

$\sqrt{36}x = 6x$

$3x + 6x - 20y\sqrt{5} + 5y\sqrt{5}$

$= 9x - 15y\sqrt{5}$

e) $\sqrt{25} - \sqrt{50}z + \sqrt{100} + 16\sqrt{2}z + 5$

$\sqrt{25} = 5$

$\sqrt{50}z = \sqrt{25\times2}\,z = 5z\sqrt{2}$

$\sqrt{100} = 10$

$5 - 5z\sqrt{2} + 10 + 16z\sqrt{2} + 5$

$= 20 + 11z\sqrt{2}$

Section B

Worked Example

Expand the expression $(\mathbf{7}\boldsymbol{a} + \mathbf{5}\sqrt{\mathbf{3}})(\mathbf{17} + \mathbf{4}\sqrt{\mathbf{3}})$

Step 1: If necessary, simplify any surds inside the brackets.

In this case we cannot simplify further, so we can go to Step 2.

Step 2: Use the FOIL method to expand the brackets.

F: $7a \times 17 = 119a$
O: $7a \times +4\sqrt{3} = +28\sqrt{3}a$
I: $+5\sqrt{3} \times 17 = +85\sqrt{3}$
L: $+5\sqrt{3} \times +4\sqrt{3} = +60$

Step 3: Collect the expanded terms.

$$(7a + 5\sqrt{3})(17 + 4\sqrt{3}) = \mathbf{119}\boldsymbol{a} + \mathbf{28}\sqrt{\mathbf{3}}\boldsymbol{a} + \mathbf{85}\sqrt{\mathbf{3}} + \mathbf{60}$$

Although we do have two constant terms and two algebraic terms, this should not be simplified further because adding the surd constant to the integer constant will give a less exact solution.

Guided Example

Expand the expression $(\mathbf{6}\boldsymbol{b} + \mathbf{6}\sqrt{\mathbf{125}})(\mathbf{18}\sqrt{\mathbf{5}} - \mathbf{10})$

Step 1: If necessary, simplify any surds inside the brackets.

$6\sqrt{125} = 6 \times \sqrt{25} \times \sqrt{5} = 30\sqrt{5}$

$(6b + 30\sqrt{5})(18\sqrt{5} - 10)$

Step 2: Use the FOIL method to expand the brackets

F : $6b \times 18\sqrt{5} = 108b\sqrt{5}$

O : $6b \times -10 = -60b$

I : $30\sqrt{5} \times 18\sqrt{5} = 30 \times 18 \times 5 = 2700$

L : $30\sqrt{5} \times -10 = -300\sqrt{5}$

Step 3: Collect the expanded terms.

$108b\sqrt{5} - 300\sqrt{5} - 60b + 2700$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Expand the following expressions:

a) $(9c + 5)(7\sqrt{3} + 4)$

F : $9c \times 7\sqrt{3} = 63c\sqrt{3}$
O : $9c \times 4 = 36c$
I : $5 \times 7\sqrt{3} = 35\sqrt{3}$
L : $5 \times 4 = 20$

$63c\sqrt{3} + 35\sqrt{3} + 36c + 20$

b) $(16 + 11\sqrt{13})(-8 + 4p)$

F : $16 \times -8 = -128$
O : $16 \times 4p = 64p$
I : $11\sqrt{13} \times -8 = -88\sqrt{13}$
L : $11\sqrt{13} \times 4p = 44p\sqrt{13}$

$64p - 128 - 88\sqrt{13} + 44p\sqrt{13}$

c) $(25 - 11z)(\sqrt{8} + 1)$

F : $25 \times \sqrt{8} = 25\sqrt{8}$
O : $25 \times 1 = 25$
I : $-11z \times \sqrt{8} = -11z\sqrt{8}$
L : $-11z \times 1 = -11z$

$25\sqrt{8} - 11z\sqrt{8} + 25 - 11z$

d) $(39 + \sqrt{16}f)(4\sqrt{2} - 9f)$ $= (39 + 4f)(4\sqrt{2} - 9f)$

$\sqrt{16}f = 4f$
F: $39 \times 4\sqrt{2} = 156\sqrt{2}$
O: $39 \times -9f = -351f$
I: $4f \times 4\sqrt{2} = 16f\sqrt{2}$
L: $4f \times -9f = -36f^2$

$156\sqrt{2} + 16f\sqrt{2} - 351f - 36f^2$

e) $(\sqrt{100} + 5\sqrt{7}x)(3\sqrt{7}x + 5y)$ $= (10 + 5x\sqrt{7})(3x\sqrt{7} + 5y)$

$\sqrt{100} = 10$

F: $10 \times 3x\sqrt{7} = 30x\sqrt{7}$
O: $10 \times 5y = 50y$
I: $5x\sqrt{7} \times 3x\sqrt{7} = 105x^2$
L: $5x\sqrt{7} \times 5y = 25xy\sqrt{7}$

$30x\sqrt{7} + 25xy\sqrt{7} + 50y + 105x^2$

Section C

Worked Example

Factorise $\sqrt{45}\boldsymbol{a} + \mathbf{16}\sqrt{\mathbf{5}}\boldsymbol{ab}$

Step 1: Reduce all surds in the expression to their simplest form.

In this case we can simplify the $\sqrt{45}$:

$$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$$

$$\sqrt{45}a + 16\sqrt{5}ab = 3\sqrt{5}a + 16\sqrt{5}ab$$

Step 2: Factorise any common factors out of the expression. Common factors could be numerical or algebraic (letters).

Initially it looks like we have no common number factors, but as we are dealing with surds we can actually take out the surd as a common factor.

So, in this case we can factor out $\sqrt{5}$ and also the $'a'$, as both terms contain this too!

$$\sqrt{\mathbf{5}}\boldsymbol{a}(\mathbf{3} + \mathbf{16}\boldsymbol{b})$$

Step 3: Expand your answer to check that it is correct. If it is correct, we will obtain the original expression.

$$\sqrt{5}a(3 + 16b) = 3\sqrt{5}a + 16\sqrt{5}ab$$

Answer: $\sqrt{\mathbf{5}}\boldsymbol{a}(\mathbf{3} + \mathbf{16}\boldsymbol{b})$

Guided Example

Factorise: $\mathbf{12}\sqrt{\mathbf{7}}\boldsymbol{c} + \sqrt{\mathbf{112}}\boldsymbol{cd}$

Step 1: Reduce all surds in the expression to their simplest form.

$\sqrt{112}\,cd = cd\sqrt{16} \times \sqrt{7} = 4cd\sqrt{7}$

$12c\sqrt{7} + 4cd\sqrt{7}$

Step 2: Factorise any common factors out of the expression. Common factors could be numerical or algebraic (letters). $-4c$ and $\sqrt{7}$

$4c\sqrt{7}(3+d)$

Step 3: Expand your answer to check that it is correct. If it is correct, we will obtain the original expression.

$4c\sqrt{7}(3+d) = 12c\sqrt{7} + 4cd\sqrt{7}$

$4c\sqrt{7}(3+d)$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Factorise the following expressions:

a) $\sqrt{52}e + 3\sqrt{13}e$

$e\sqrt{52} = e\sqrt{13\times4} = 2e\sqrt{13}$

$2e\sqrt{13} + 3e\sqrt{13} = 5e\sqrt{13}$

$5e\sqrt{13}$

b) $12\sqrt{12}st + \sqrt{12}s$

$\sqrt{12} = \sqrt{3\times4} = 2\sqrt{3}$

$s\sqrt{12}(12t+1)$

$2s\sqrt{3}(12t+1)$

c) $\sqrt{44}ab + 8\sqrt{11}abc$

$\sqrt{44} = \sqrt{11\times4} = 2\sqrt{11}$

$2\sqrt{11}ab + 8\sqrt{11}abc$ — $2\sqrt{11}ab$ is common

$2ab\sqrt{11}(1+4c)$ $\quad \dfrac{8\cancel{\sqrt{11}}\cancel{ab}c}{2\cancel{\sqrt{11}}\cancel{ab}}$

d) $\sqrt{180}xy + \sqrt{45}x + 8\sqrt{5}xyz$

$\sqrt{180} = \sqrt{36\times5} = 6\sqrt{5}$

$\sqrt{45} = \sqrt{9\times5} = 3\sqrt{5}$

$6xy\sqrt{5} + 3x\sqrt{5} + 8xyz\sqrt{5}$ $\quad x\sqrt{5}$ is common

$x\sqrt{5}(6y+3+8yz)$

e) $3\sqrt{17}jk + \sqrt{68}jk + 5\sqrt{17}jklm + \sqrt{153}j$

$\sqrt{68} = \sqrt{4\times17} = 2\sqrt{17}$

$\sqrt{153} = \sqrt{9\times17} = 3\sqrt{17}$

$3jk\sqrt{17} + 2jk\sqrt{17} + 5jklm\sqrt{17} + 3j\sqrt{17}$ $\quad j\sqrt{17}$ is common

$j\sqrt{17}(3k+2k+5klm+3)$

$= j\sqrt{17}(5k+5klm+3)$

Section D – Higher Only

Worked Example

Simplify the expression $\frac{27a^4b^3}{3ab}$

Step 1: Cancel constants from the numerator and denominator.

In this case we will divide both the numerator and denominator by 3:

$$\frac{27a^4b^3}{3ab} = \frac{9a^4b^3}{ab}$$

Step 2: Cancel any algebra (letter) terms from the numerator and denominator. Use index laws to work out how the powers will be affected.

In this case will divide both the numerator and denominator by $'a'$*. This leaves us with:*

$$\frac{9a^4b^3}{ab} = \frac{9a^3b^3}{b}$$

Then we will divide both the numerator and denominator by $'b'$*. This leaves us with:*

$$\frac{9a^3b^3}{b} = 9a^3b^2$$

Answer: $\boldsymbol{9a^3b^3}$

Guided Example

Simplify the expression $\frac{54c^8d^4}{9c^2d^3}$

Step 1: Cancel constants from the numerator and denominator.

$$\frac{54c^8d^4}{9c^2d^3} \underset{\div 9}{\overset{\div 9}{=}} \frac{6c^8d^4}{c^2d^3}$$

Step 2: Cancel any algebra (letter) terms from the numerator and denominator. Use index laws to work out how the powers will be affected.

$$\frac{6c^8d^4}{c^2d^3} \underset{\div c^2d^3}{\overset{\div c^2d^3}{=}} 6c^6d$$

c^{8-2}

d^{4-3}

$6c^6d$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Simplify the following expressions:

a) $\frac{16x^2y}{4xy}$ $\underset{\div 4}{\overset{\div 4}{=}}$ $\frac{4x^2y}{xy}$

$\frac{4x^2y}{xy}$ $\underset{\div xy}{\overset{\div xy}{=}}$ $4x$ — x^{2-1}

$4x$

b) $\frac{31a^2b^2}{4ab}$

$\frac{31a^2b^2}{4ab} = \frac{31ab}{4}$ — $a^2 \div a$, $b^2 \div b$

$\frac{31ab}{4}$

c) $\frac{42p^8q^7}{6p^6q^5}$ $\underset{\div 6}{\overset{\div 6}{}}$ $\frac{7p^8q^7}{p^6q^5}$

$\frac{7p^8q^7}{p^6q^5}$ $\underset{\div p^6q^5}{\overset{\div p^6q^5}{=}}$ $7p^2q^2$ — p^{8-6}, q^{7-5}

$7p^2q^2$

d) $\frac{100e^4f^2g}{10e^2f}$ $\underset{\div 10}{\overset{\div 10}{}}$ $\frac{10e^4f^2g}{e^2f}$

$\frac{10e^4f^2g}{e^2f}$ $\underset{}{\overset{\div e^2f}{=}}$ $10e^2fg$ — e^{4-2}, f^{2-1}

$10e^2fg$

e) $\frac{77j^5k^7l^9}{7j^2k^4l^6}$ $\underset{\div 7}{\overset{\div 7}{}}$ $\frac{11j^5k^7l^9}{j^2k^4l^6}$

$\frac{11j^5k^7l^9}{j^2k^4l^6}$ $\overset{\div j^2k^4l^6}{=}$ $11j^3k^3l^3$

Section E – Higher Only

Worked Example

Simplify the expression $\frac{x^2-25}{x^2+9x+20}$

Step 1: Factorise any quadratic expressions.

$$\frac{x^2-25}{x^2+9x+20} = \frac{(x+5)(x-5)}{(x+5)(x+4)}$$

Step 2: Cancel any factorised terms which appear in the numerator and the denominator.

We can simplify the fraction as both the numerator and denominator contain an $(x+5)$ *term being multiplied by another term.*

We simplify the fraction by dividing by $(x+5)$ *term on the top and bottom, leaving us with:*

$$\frac{x^2-25}{x^2+9x+20} = \frac{(x+5)(x-5)}{(x+5)(x+4)} = \frac{(x-5)}{(x+4)}$$

Answer: $\frac{(x-5)}{(x+4)}$

Guided Example

Simplify the expression $\frac{x^2-64}{(x+2)(x+8)}$

Step 1: Factorise any quadratic expressions.

$x^2 - 64$ difference of two squares

$= (x-8)(x+8)$

Step 2: Cancel any factorised terms which appear in the numerator and the denominator.

$$\frac{(x-8)\cancel{(x+8)}}{(x+2)\cancel{(x+8)}}$$

$$\frac{x-8}{x+2}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Simplify the following:

a) $\frac{(x+2)(x+1)}{x^2+5x+6}$

$x^2 + 5x + 6$ x to 6 + to 5 2 and 3

$= (x+2)(x+3)$

$\frac{(\cancel{x+2})(x+1)}{(\cancel{x+2})(x+3)} = \boxed{\frac{(x+1)}{(x+3)}}$

b) $\frac{y^2+2y-3}{y^2+7y+12}$

$y^2 + 2y - 3 = (y+3)(y-1)$

$y^2 + 7y + 12 = (y+3)(y+4)$

$\frac{(\cancel{y+3})(y-1)}{(\cancel{y+3})(y+4)} = \boxed{\frac{(y-1)}{(y+4)}}$

c) $\frac{a^2-49}{a^2+12a+35}$

$a^2 - 49 = (a+7)(a-7)$

$a^2 + 12a + 35 = (a+7)(a+5)$

$\frac{(\cancel{a+7})(a-7)}{(\cancel{a+7})(a+5)} = \boxed{\frac{(a-7)}{(a+5)}}$

d) $\frac{b^2-9b+18}{b^2-9}$

$b^2 - 9b + 18 = (b-6)(b-3)$

$b^2 - 9 = (b+3)(b-3)$

$\frac{(b-6)(\cancel{b-3})}{(b+3)(\cancel{b-3})} = \boxed{\frac{(b-6)}{(b+3)}}$

e) $\frac{(z^2-1)(z+3)}{(z+1)(z+5)(z-1)}$

$z^2 - 1 = (z+1)(z-1)$

$\frac{(\cancel{z-1})(\cancel{z+1})(z+3)}{(\cancel{z-1})(\cancel{z+1})(z+5)} = \boxed{\frac{(z+3)}{(z+5)}}$

Section F – Higher Only

Worked Example

Simplify the expression $\frac{x^2-64}{6} \times \frac{12}{(x+8)(x+2)}$

Step 1: Simplify the fractions by factorising expressions and dividing by any common terms.

First, we can simplify $x^2 - 64$ *(as it is the difference of two squares) into* $(x+8)(x-8)$.

$$\frac{x^2-64}{6} \times \frac{12}{(x+8)(x+2)} = \frac{(x+8)(x-8)}{6} \times \frac{12}{(x+8)(x+2)}$$

Each individual fraction cannot be simplified further, but cross simplification, just like with normal fractions, can be used. We can cancel the $(x+8)$ *terms by dividing both fractions by* $(x+8)$.

$$\frac{(x+8)(x-8)}{6} \times \frac{12}{(x+8)(x+2)} = \frac{(x-8)}{6} \times \frac{12}{(x+2)}$$

Lastly, cross simplification can be used to cancel the constant terms of 6 *and* 12 *as they both have common factors, and so can both be divided by* 6.

$$\frac{(x-8)}{6} \times \frac{12}{(x+2)} = \frac{(x-8)}{1} \times \frac{2}{(x+2)}$$

Step 2: Separately multiply the numerators together and the denominators together.

$$\frac{(x-8)}{1} \times \frac{2}{(x+2)} = \mathbf{\frac{2(x-8)}{(x+2)}}$$

Guided Example

Simplify the expression $\frac{5}{x^2-25} \times \frac{(x+5)(x+2)}{17}$

Step 1: Simplify the fractions by factorising expressions and dividing by any common terms.

$x^2 - 25 = (x+5)(x-5)$

$$\frac{5}{\cancel{(x+5)}(x-5)} \times \frac{\cancel{(x+5)}(x+2)}{17} = \frac{5}{(x-5)} \times \frac{(x+2)}{17}$$

Step 2: Separately multiply the numerators together and the denominators together.

$$\frac{5(x+2)}{17(x-5)}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

6. Simplify the following:

a) $\frac{(x+3)}{3} \times \frac{6}{(x+2)}$

$= \frac{(x+3)}{1\cancel{\times 3}} \times \frac{2\cancel{\times 3}}{(x+2)} = \boxed{\frac{2(x+3)}{(x+2)}}$

b) $\frac{8}{2\cancel{(y+2)}} \times \frac{\cancel{(y+2)}}{(y+8)}$

$= \frac{\cancel{2}\times 4}{\cancel{2}} \times \frac{1}{(y+8)} = \boxed{\frac{4}{(y+8)}}$

c) $\frac{7}{(a+9)} \times \frac{a^2-81}{14}$

$a^2-81 = (a+9)(a-9)$

$\frac{\cancel{7}}{\cancel{a+9}} \times \frac{\cancel{(a+9)}(a-9)}{\cancel{7}\times 2} = \boxed{\frac{(a-9)}{2}}$

d) $\frac{b^2-36}{5(b+6)} \times \frac{11}{(b-6)}$

$b^2-36 = (b+6)(b-6)$

$\frac{\cancel{(b+6)}\cancel{(b-6)}}{5\cancel{(b+6)}} \times \frac{11}{\cancel{(b-6)}} = \boxed{\frac{11}{5}}$

e) $\frac{z^2-3z-10}{(z+4)} \times \frac{z^2-16}{6(z+2)}$

$z^2-3z-10 = (z-5)(z+2)$
$z^2-16 = (z-4)(z+4)$

$\frac{(z-5)\cancel{(z+2)}}{\cancel{(z+4)}} \times \frac{(z-4)\cancel{(z+4)}}{6\cancel{(z+2)}} = \boxed{\frac{(z-5)(z-4)}{6}}$

Section G

Worked Example

Simplify the expression $\frac{7}{x^2} \div \frac{x}{9}$

Step 1: When dividing two fractions, including algebraic fractions, flip the second fraction and then multiply them together.

$$\frac{7}{x^2} \div \frac{x}{9} = \frac{7}{x^2} \times \frac{9}{x}$$

Step 2: Try to simplify the fractions by factorising any expressions which can be factorised. Divide out by any common terms to cancel them.

$$\frac{7}{x^2} \times \frac{9}{x}$$

This answer cannot be simplified further.

Step 3: Separately multiply the numerators together and the denominators together.

$$\frac{7}{x^2} \times \frac{9}{x} = \frac{7 \times 9}{x^2 \times x} = \frac{\mathbf{63}}{\boldsymbol{x^3}}$$

This answer cannot be simplified further.

Guided Example

Simplify the expression $\frac{5}{x} \div \frac{10x^9}{3}$

Step 1: When dividing two fractions, including algebraic fractions, flip the second fraction and then multiply them together.

$$\frac{5}{x} \times \frac{3}{10x^9}$$

Step 2: Try to simplify the fractions by factorising any expressions which can be factorised. Divide out by any common terms to cancel them. Common factor of 5

$$\frac{1}{x} \times \frac{3}{2x^9}$$

Step 3: Separately multiply the numerators together and the denominators together.

$$\frac{3}{2x^{10}}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

7. Simplify the following:

a) $\frac{3}{x} \div \frac{x}{4}$

$\frac{3}{x} \times \frac{4}{x} = \frac{12}{x^2}$

b) $\frac{19}{a^3} \div \frac{a^7}{38}$

$\frac{19}{a^3} \times \frac{38}{a^7} = \frac{722}{a^{10}}$

a^{3+7}

c) $\frac{17}{4b^4} \div \frac{3b^4}{8}$

$(2 \times \cancel{4})$

$\frac{17}{\cancel{4}b^4} \times \frac{8}{3b^4} = \frac{34}{3b^8}$

d) $\frac{13(x-3)}{x^2-9} \div \frac{5}{(x+3)}$

$x^2-9 = (x+3)(x-3)$

$= \frac{13\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x-3)}} \times \frac{\cancel{(x+3)}}{5} = \frac{13}{5}$

e) $\frac{(z+9)}{z^2+3z+2} \div \frac{z^2-9}{(z+1)}$

$z^2-9 = (z-3)(z+3)$
$z^2+3z+2 = (z+1)(z+2)$

$\frac{(z+9)}{(\cancel{z+1})(z+2)} \times \frac{\cancel{(z+1)}}{(z-3)(z+3)}$

$= \frac{(z+9)}{(z+2)(z^2-9)}$

Section H – Higher Only

Worked Example

Write $\frac{5}{(x+1)} + \frac{4}{(x+2)}$ as a single fraction.

Step 1: Work out what the common denominator will be. The common denominator can be found by multiplying the two denominators together.

The common denominator in this example will be $(x+1)(x+2)$.

Step 2: Re-write each fraction with the common denominator.

For each fraction, multiply the numerator and denominator by the same value that is required to get the denominator in the form of the common denominator $(x+1)(x+2)$.

This means we will need to multiply the numerator and denominator of the first fraction by $(x+2)$ *and we will need to multiply the denominator and numerator of the second fraction by* $(x+1)$*:*

$$\frac{5}{(x+1)} + \frac{4}{(x+2)} = \frac{5(x+2)}{(x+1)(x+2)} + \frac{4(x+1)}{(x+2)(x+1)} = \frac{5(x+2)+4(x+1)}{(x+1)(x+2)}$$

Step 3: Simplify the numerator by expanding and collecting like terms.

$$\frac{5(x+2)+4(x+1)}{(x+1)(x+2)} = \frac{5x+10+4x+4}{(x+1)(x+2)} = \mathbf{\frac{9x+14}{(x+1)(x+2)}}$$

Guided Example

Write $\frac{7}{(x+3)} + \frac{3}{(x+2)}$ as a single fraction.

Step 1: Work out what the common denominator will be. The common denominator can be found by multiplying the two denominators together.

Common denominator : $(x+3)(x+2)$

Step 2: Rewrite each fraction over the common denominator.

$$\frac{7(x+2)+3(x+3)}{(x+3)(x+2)}$$

Step 3: Simplify the numerator by expanding and collecting like terms.

$$\frac{7x+14+3x+9}{(x+3)(x+2)} = \frac{10x+23}{(x+3)(x+2)}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

8. Write the following expressions as single fractions.

Butterfly method

a) $\frac{3}{(x+1)}+\frac{8}{(x+7)}$

common denominator: $(x+1)(x+7)$

$$\frac{3(x+7)+8(x+1)}{(x+1)(x+7)}=\frac{3x+21+8x+8}{(x+1)(x+7)}$$

$$=\frac{11x+29}{(x+1)(x+7)}$$

Butterfly method

b) $\frac{6}{(y+2)}-\frac{9}{(y+3)}$

common denominator: $(y+2)(y+3)$

$$\frac{6(y+3)-9(y+2)}{(y+2)(y+3)}=\frac{6y+18-9y-18}{(y+2)(y+3)}$$

$$=\frac{-3y}{(y+2)(y+3)}$$

Butterfly method

c) $\frac{7}{(a+5)}+\frac{8}{(a-3)}$

common denominator: $(a+5)(a-3)$

$$\frac{7(a-3)+8(a+5)}{(a+5)(a-3)}=\frac{7a-21+8a+40}{(a+5)(a-3)}$$

$$=\frac{15a+19}{(a+5)(a-3)}$$

Butterfly method

d) $\frac{6}{(b-2)}-\frac{7}{(b+4)}$

common denominator: $(b-2)(b+4)$

$$\frac{6(b+4)-7(b-2)}{(b-2)(b+4)}=\frac{6b+24-7b+14}{(b-2)(b+4)}$$

$$=\frac{38-b}{(b-2)(b+4)}$$

Butterfly method

e) $\frac{4}{(z-9)}-\frac{7}{(z-5)}$

common denominator: $(z-9)(z-5)$

$$\frac{4(z-5)-7(z-9)}{(z-9)(z-5)}=\frac{4z-20-7z+63}{(z-9)(z-5)}$$

$$=\frac{43-3z}{(z-9)(z-5)}$$